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ONE-DIMENSIONAL SEPARATION MODEL

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The current literature on removal of dust from gas employs a separation model [1] that necessitates the use of experimental data on the fractional separation efficiency. The present study considers a physical model that enables determination of the separation efficiency without resorting to experimental data.

Analyzing the processes of cleaning dust-laden gas flows shows that cleaning occurs under transverse action on a particle (see Fig. 1), as a result of which the particle shifts normally to the flow. A transverse effective force, for example, a gravity force, a centrifugal force, or an electric force, would cause a particle to move with a transverse acceleration a in the absence of other forces. If m_p is the particle mass, then this force is related to the acceleration by $F_m = m_p a$. Analyzing all forces acting on a particle in the flow reveals the need for taking into account the pressure-gradient force $F_p = V_p a \rho$ and the aerodynamic-resistance force in the form of the Stokes law $F_a = 3\pi\mu d_a u_p$. Then, the equation for the transverse motion of a particle takes the form

$$m_p \frac{du_p}{dt} = m_p a - 3\pi\mu d_a u_p - V_p \rho a, \qquad (1)$$

where a is the transverse mass separation force based on the particle mass.

By integrating the linear differential equation (1) with the boundary condition $u_p(0) = 0$ we obtain

$$u_p = \frac{a m_p (1 - \bar{\rho})}{3\pi\mu d_a} (1 - \exp(-kt)), \qquad (2)$$

where

$$k = \frac{3\pi\mu d_a}{m_p}.$$

On reintegrating expression (2) we determine the coordinate x travelled by a particle:

$$x = \frac{a m_p (1 - \bar{\rho})}{3\pi\mu d_a} \left(t + \frac{\exp(-kt) - 1}{k} \right), \qquad (3)$$

provided the particle starts to move from the coordinate origin (see Fig. 1).

We consider a flow uniformly dust-laden by identical particles of concentration C_{in}. If the particles reach the lower plane in Fig. 1 by the action of the force F_m, they will be regarded as separated from the flow. At the outlet from a channel of length l, the upper part of the flow of width x will be free of particles. Therefore, the mass flow rate of the separated particles per unit channel width is

$$M_x = C_{in} v_l x,$$

where v_l is the flow velocity along the channel. Because the mass flow rate of particles at the channel inlet is $M_{in} = C_{in} v_l b$, the efficiency of separation of the flow from monodispersed particles of diameter d is

$$\eta_d = \frac{M_x}{M_{in}} = \frac{x}{b}.$$

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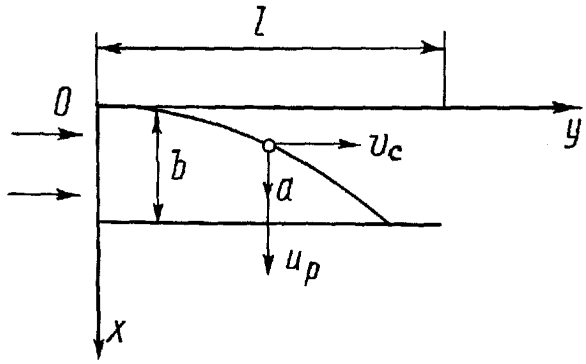

Fig. 1. One-dimensional separation model.

Since over the time t the flow moves the distance $l = v_l t$, in view of Eq. (3) the fractional degree of cleaning in a plane flow with the transverse force F_m is written as

$$\eta_d = K_s^2 \left(1 - \frac{\exp(-S_p) - 1}{S_p}\right), \tag{4}$$

where

$$S_p = \frac{3\pi\mu d_a l}{m_p v_l}; \tag{5}$$

$$K_s = \sqrt{\frac{(1-\bar{\rho})\, a l^2}{S_p v_l^2 b}}; \tag{6}$$

$\bar{\rho} = \rho/\rho_p$ is the relative density.

The quantity S_p is a dimensionless hydrodynamic parameter of the of action the flow on a particle. The parameter K_s will be called the separation coefficient. It should be noted that expression (4) for particles lighter than the carrying medium ($\bar{\rho} > 1$) is negative. This means that a particle moves in the direction opposite to the acceleration a (see Fig. 1). Therefore, such particles, separated from the flow, accumulate at the upper plate.

Real two-phase flows contain particles of various diameters. Let the disperse composition be defined by the integral distribution function

$$D = \frac{M_d}{M}, \tag{7}$$

where M_d is the mass flow rate of particles of diameter equal to or smaller than d; and M is the mass flow rate of all particles.

The distribution function D is also called a pass. The mass flow rate of the particles of diameter d within the deviation Δd is

$$\Delta M = M \frac{dD}{dd} \Delta d.$$

With the fractional separation efficiency η_d the flow rate of separated particles is

$$\Delta M_s \doteq M \eta_d \frac{dD}{dd} \Delta d.$$

Integrating this expression over all diameters of the particles we obtain the flow rate of the trapped particles M_s. Then the separation efficiency for a polydispersed flow is written as

$$\eta = \frac{M_s}{M} = \int_0^\infty \eta_d \frac{dD}{dd} dd. \tag{8}$$

For many dusts, for example, for light ashes at the outlet from boiler plants [2], the log-normal distribution

$$D = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\tau} \exp(-0{,}5\tau^2)\, d\tau, \tag{9}$$

is valid, where

$$\tau = \lg\left(\frac{d}{d_{50}}\right) \Big/ \lg \sigma,$$

d_{50} is the median diameter of the distribution , at which the mass of the particles with $d < d_{50}$ is equal to the mass of the particles with $d > d_{50}$.

Thus, the disperse composition with the distribution law (9) is characterized by two parameters, viz., d_{50} and σ. At a certain particle diameter $d = d_{cr}$, the value of η_d becomes equal to unity and does not change with a further increase in d_p. Therefore, the integration range in Eq. (8) can be subdivided into two intervals, namely, $0 \le d \le d_{cr}$ and $d_{cr} \le d < \infty$. On the second interval

$$\int_{d_{cr}}^{\infty} \frac{dD}{dd}\, dd = 1 - D(\tau_{cr}),$$

where d_{cr} is determined from Eq. (4) at $\eta_d = 1$ and $D(\tau_{cr})$ is the mass of the particles of diameters $0 \le d < d_{cr}$.

Then, with allowance for Eq. (9), the separation efficiency for the polydisperse flow is

$$\eta = 1 - D(\tau_{cr}) + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\tau_{cr}} \eta_d \exp(-0{,}5\tau^2)\, d\tau, \tag{10}$$

where

$$\tau_{cr} = [\lg(d_{cr}/d_{50})]/\lg\sigma.$$

Expression (10) is obtained theoretically and involves all hydrodynamic parameters on which the cleaning process depends.

Equation (4) and, therefore, Eq. (10) allow calculation of the separation efficiency when forces of various nature act on the particles, namely, gravitational, inertial, centrifugal, and electromagnetic. For example, in the case of electrostatic forces, their magnitude can be taken into account in terms of the acceleration $a = F_e/m$, where $F_e = qE$, q is the particle charge, and E is the electrostatic strength. In the case of electric forces, account must be taken of the fact that they do not act on the gas flow and do not produce a pressure gradient; therefore, $\bar{p} = 0$ must additionally be taken in Eq. (6).

As a case in point we consider the separation efficiency for spherical Stokes particles $d_a = d_p$. Then, with consideration of Eqs. (4)-(6), we obtain

$$S_p = \frac{18\mu l}{\rho_p d_p^2 v_l}; \quad \eta_d \approx (K_s)^2, \tag{11}$$

and the condition $\eta_d = 1$ gives

$$d_{cr} = \sqrt{\frac{18\mu v_l b}{(1-\bar{\rho})\, a l \rho_p}}.$$

Based on these data, the third term in Eq. (10) transforms to

$$\frac{K_s^2(d_{50}) \exp(2\ln^2\sigma)}{\sqrt{2\pi}} \int_{-\infty}^{\xi_{cr}} \exp(-0{,}5\xi^2)\, d\xi = K_s^2(d_{50}) \exp(2\ln^2\sigma)\, D(\xi_{cr}), \tag{12}$$

where $K_s(d_{50})$ is the separation coefficient at $d_p = d_{50}$;

$$\xi = \tau - 2\ln\sigma; \quad \xi_{cr} = \tau_{cr} - 2\ln\sigma. \tag{13}$$

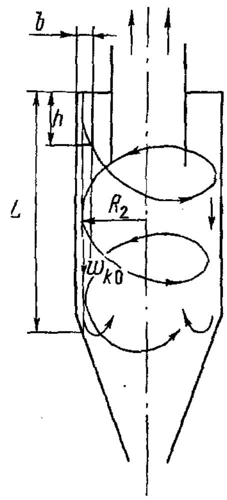

Fig. 2. Parameters of the one-dimensional model of separation in a cyclone.

Here, the transformation $d^2 = d^2_{50}\exp(2\ln\sigma\tau)$ was used in the derivation. Then, the expression for the separation efficiency of the polydisperse flow of spherical Stokes particles is written as

$$\eta = 1 - D(\tau_{cr}) + K_s(d_{50})D(\xi_{cr})\exp(2\ln^2\sigma). \qquad (14)$$

The function D of the arguments τ_{cr}, ξ_{cr}, in conformity with Eq. (9), is the probability integral $\Phi(x)$, whose values are tabulated and can be determined as a function of the arguments.

We apply the results of this model to identify the separation efficiency for a cyclone. In accordance with a flow in cyclones and vortex chambers [3], it can be assumed (see Fig. 2) that the flow, which entered the cyclone through an inlet connecting pipe of cross section h × b, extends downward in the form of an annular rotating jet of width b. We will consider all particles that reach the cyclone wall by the action of the centrifugal force to then be trapped. On turning within the lower part of the cyclone the cleaned gas flow will start moving upward to the outlet branch pipe. Separation in the ascending flow is disregarded.

The characteristics of such a rotary annular flow are as follows. If R_2 is the mean radius of the annulus and b is the width of the inlet branch pipe, then the mean flow-rate velocity in the annulus is $w_{an} = G/(2\pi R_2 b\rho)$. The mean tangential velocity at the radius R_2 is $v_2 = v_{fr}R_{fr}/R_2$, the total flow velocity is $v_l = (v_2^2 + w_{an}^2)^{1/2}$, and the centrifugal acceleration is $a = v_2^2/R_2$. Because the time of the flow along the cyclone length L is $t = L/w_{an}$, the flow will travel the whole path, equal to $l = v_l t = L(1 + v_2^2/w_{an}^2)^{1/2}$. These characteristics make it clear that the rotating annular flow can be developed into the plane (according to Fig. 1) in which the particle separation occurs by the action of the centrifugal force directed transversely to the flow. After l, v_l, and a have been substituted into Eqs. (5) and (6) for S_p and K_s, the expressions obtained for the separation efficiency are applicable to the cyclone dust separator. They allow determination of all geometric and hydrodynamic factors for any cyclone.

Based on the above technique, the computer program STOCh was developed, which determines the separation efficiency of the cyclone. The calculation requires that the separator dimensions, the physical properties of the medium and the particles, the volumetric flow rate of the medium, and the disperse dust composition be specified.

To validate the technique, separation efficiencies of cyclones η_p were calculated, for which various authors, mainly D. S. Sazhin and his colleagues, obtained experimental separation efficiencies η_e. The results are compared in Table 1, where it is evident that the separation efficiencies predicted by the program STOCh fit the experimental data and reflect well the influence of the gas flow rate and the particle diameter on the cleaning efficiency. It should be noted that this method can be used to design new cyclone dust separators with prescribed characteristics or to improve the efficiency of those currently available.

TABLE 1. Comparison of Calculated η_p and Measured η_0 Separation Efficiencies

Volumetric flow rate of gases, m^2/h	Cyclone diameter D_{fr}, m	Particle density ρ_p, kg/m^3	Median diameter d_{50}, μm	$\lg\sigma$	η_p	η_e	Reference
2120	0.5	2650	30	0.519	95.3	95.6	[4]
2400	0.5	2650	30	0.519	95.7	96	[4]
2800	0.5	2650	30	0.519	96.3	96.4	[4]
2470	0.5	2650	20	0.525	91.7	86.1	[4]
2470	0.5	2650	10	0.561	78.5	79.5	[4]
11304	1.0	2200	35.8	0.312	98.8	97.3	[6]
11304	1.0	2300	19.1	0.46	84	86.2	[6]
6500	0.8	2650	10	0.561	71	72.5	[5]

NOTATION

a, particle acceleration due to the separation force; b, channel width; C_{in}, particle concentration at the inlet; d_a, d_{cr}, d_{50}, aerodynamic, critical, and median particle diameters; D, distribution function (pass); E, electric field strength; F_m, F_p, F_a, forces acting on the particle; G, mass flow rate of the carrying medium; K_s, separation coefficient; l, L, channel and cyclone lengths; m_p, particle mass; M, M_s, M_x, M_{in}, mass flow rates of the particles; R_{fr}, R_2, cyclone radius and predicted radius; S_p, parameter of the effect of the flow on a particle; t, time; u_p, transverse particle velocity; v_l, flow velocity along the channel; v_{fr}, tangential velocity at the cyclone wall; v_2, mean tangential flow velocity; V_p, particle volume; w_{an}, flow velocity in the annulus; x, transverse coordinate; y, longitudinal coordinate; η, η_d, total and fractional separation efficiencies; μ, dynamic viscosity of the medium; σ, dispersion; ρ, ρ_p, medium and particle densities; τ, τ_{cr}, ξ_{cr}, arguments of the distribution function.

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